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Reciprocal Relationships Between Attitude Toward Mathematics and Achievement in Mathematics

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ABSTRACT **Mathematics educators have done little to investigate the reciprocal relationship between attitude toward mathematics and achievement in mathematics. In this study, the reciprocal relationship was modeled after LISREL, using data from a Dominican national evaluation of high school mathematics (*N* = 1,044). Three data sets that were used to examine a hypothesized causal model demonstrated relatively good results on model-data-fit. Major findings from the model included: (a) A reciprocal relationship existed between every attitudinal measure and mathematics achievement. (b) The feeling of enjoyment, not the feeling of difficulty, directly affected mathematics achievement. (c) The feeling of difficulty functioned via the feeling of enjoyment to affect mathematics achievement. (d) The perception of mathematics as important was independent of other attitudinal measures. The findings suggest that the reciprocal or interactive nature between attitude toward mathematics and achievement in mathematics can substantially modify their casual relationship. A unilateral relationship is likely to overestimate the causal effect between attitude toward mathematics and achievement in mathematics.**

A causal relationship between attitude toward mathematics (ATM) and achievement in mathematics (AIM) has long been assumed to exist. That is, a more positive ATM contributes to a higher level of AIM (Suydam & Weaver, 1975). Research studies indicate that ATM plays an important role in explaining AIM (e.g., Ethington & Wolfle, 1984, 1986; Lester, Garofalo, & Kroll, 1989; Loebl, 1993; Marshall, 1989; Sherman, 1980). Schoenfeld (1985) and Silver (1985) demonstrated that students' ATM affects their mathematical abilities to solve nonroutine problems.

However, some methodological concerns have plagued research on the ATM–AIM relationship. Most studies use correlation coefficients as measures of the relationship and therefore do not provide clear evidence in regard to whether ATM is a cause or an effect of AIM (Enemark & Wise, 1981; Neale, 1969). Quinn and Jadav (1987) argued that distinctions ought to be made between a symmetrical ATM–AIM relationship and a causal ATM–AIM relationship. Although researchers have published studies on the causal relationship between ATM and AIM (e.g., Anderson, 1981; Ethington & Wolfle, 1984, 1986; Loebl, 1993; Quinn & Jadav, 1987; Randhawa, Beamer, & Lundberg, 1992; Robinson, 1975; Wolf & Blixt, 1981), they tended to conduct them from a unidirectional perspective. For example, Ethington and Wolfle (1984, 1986) proposed structural equation models of mathematics achievement in which ATM is specified to cause AIM. Keeves (1986), however, in his investigation of the performance cycle, suggested that the main causal chain should be hypothesized as "initial achievement and initial attitudes → academic motivation → attentiveness → final achievement → final attitudes" (p. 148).

The concern is not about the inconsistency in the causal direction between ATM and AIM but, rather, the appropriateness of specifying a unilateral relationship between ATM and AIM. Ethington and Wolfle (1986) discussed the relationship between mathematics attitude and exposure:

> Although it might be argued that enrollment in mathematics courses is likely to affect attitudes toward mathematics, an equally plausible argument may be made that these attitudes affect decisions to enroll in mathematics courses. Thus, specifying any unidirectional causal relationship between these factors would be inappropriate. (p. 66)

Similar concerns have motivated some researchers to recognize the need to consider *bilateral relationships.* For example, Feather (1988) indicated a reciprocal relationship between mathematics ability and mathematics valence. In their model of mathematics anxiety, Meece, Wigfield, and Eccles (1990) tested a bidirectional relationship between expectancies of mathematics performance and importance ratings students attach to mathematics.

It seems reasonable to suspect that ATM and AIM affect each other. First, the results of path analyses have shown similar path correlation coefficients between ATM on AIM

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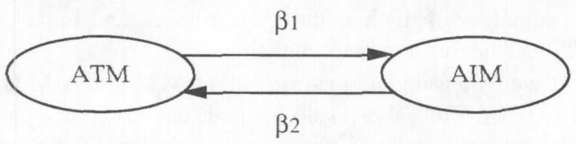
and AIM on ATM (e.g., Anderson, 1981; Keeves, 1986). It is therefore questionable to consider a unilateral relationship between ATM and AIM. Second, some researchers (see McLeod, 1992) believe that neither attitude nor achievement depends on the other; rather, they interact with each other in a complicated manner. A reciprocal relationship seems to be able to capture the interactive nature of ATM and AIM.

The reciprocal relationship between ATM and AIM is denoted as *loop* between ATM and AIM (see Hayduk, 1987, for a discussion of this concept). Unlike unilateral relationships, the unilateral effect of, say, ATM on AIM is modified either positively or negatively by the interaction between ATM and AIM in reciprocal relationships. In the literature of structural equation modeling, this modification is referred to as *loop enhancement.* Loop enhancement is calculated as $1/(1 - L)$, where L is the coefficient product of those paths that form a causal loop (Hayduk, 1987). The reciprocal relationship between ATM and AIM is graphically depicted in Figure 1. The modified direct or indirect effect is simply the product of the ordinary effect and the loop enhancement $1/(1 - L)$. A direct effect is an unmediated influence of one variable on the other; a medium variable is needed to establish an indirect effect of one variable on the other, for example, ATM may affect AIM through mathematics participation (Anderson, 1981).

Researchers primarily investigated mediating variables independent of ATM and AIM. For example, Anderson (1981) found that the effect of ATM on AIM is modified by mathematics participation. Few researchers, however, have noted that the effect of ATM on AIM can also be modified by the loop enhancement of their own. If the loop enhancement is greater than 1, it strengthens the effect of ATM on AIM or that of AIM on ATM (the modified effect is the product of the normal effect and the loop enhancement). Remedial programs on ATM have little impact on AIM, and high AIM does not necessarily lead to favorable ATM, if their loop enhancement is substantially smaller than 1. Furthermore, although path coefficients may indicate a positive effect of AIM on ATM or ATM on AIM, the modified result is actually negative if their loop enhancement is negative. Therefore, the reciprocal relationship between ATM and AIM is informative both theoretically and practically.

Figure 1. The Loop and Loop-Enhancement Coefficient Depicting the Reciprocal Relationship Between ATM and AIM



Loop enhancement = $1/(1 - L)$, where $L = \beta1\beta2$
Modified effect of ATM on AIM = $(\beta1)/(1 - \beta1\beta2)$
Modified effect of AIM on ATM = $(\beta2)/(1 - \beta1\beta2)$

Using a national sample of high school seniors from the Dominican Republic, I tested the reciprocal relationship between ATM and AIM by examining their loop enhancement and its impact on the effect of ATM on AIM and that of AIM on ATM in three structural equation models. The major research questions were:

1. What structural model best describes the reciprocal relationship between ATM and AIM?
2. What role does each selected variable play in the model with respect to the reciprocal relationship between ATM and AIM?

Researchers often assume different definitions of ATM (McLeod, 1992). For example, Leder (1987) and Reyes (1984) used ATM as a general concept that includes beliefs about self and about mathematics. The multiplicity of meaning given to the concept of ATM is the primary culprit of the inconsistencies in the literature on ATM (Anderson, 1981). A reasonable solution is to measure attitude toward specific mathematical activities rather than a "generalized attitude" toward mathematics (Aiken, 1970b). Hence, I defined ATM as either positive or negative responses, in terms of importance, difficulty, and enjoyment , when learning algebra, geometry, and trigonometry. This definition of ATM allowed an examination of whether feelings about mathematics as important, difficult, and enjoyable causally linked with one another, and with AIM, in a similar way across the three mathematical areas.

Method

Sample

Participants in the present study were high school seniors from the Dominican Republic. The data were collected during the 1988–89 school year in a national evaluation project on learning high school mathematics. The Dominican research team obtained a national sample of 1,200 high school senior students through stratified random sampling (See Luna & Gonzalez, 1988). The sample was considered representative in terms of geographical region and school organization. Subjects were administered one student questionnaire and two mathematics achievement tests. Students with missing values were deleted from the national sample, resulting in a sample of 1,044 high school seniors for analysis in this study. List-wise deletion of students with missing values should not produce serious biases for the following statistical analyses, because missing values scatterred randomly in the sample distribution.

Measures

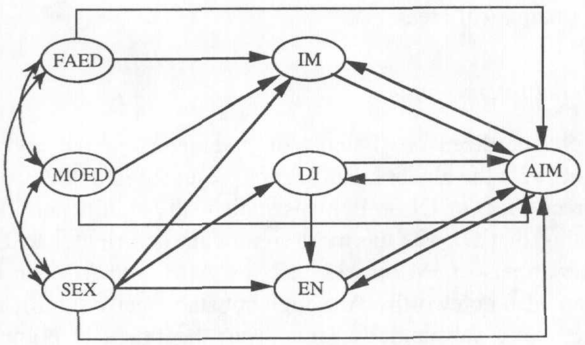
The two mathematics achievement tests contained 70 multiple-choice items (35 on each test) that covered four major curriculum strands in mathematics: arithmetic, algebra, geometry, and trigonometry. The cognitive skills required to solve problems included computation, comprehension, and application (See Luna & Gonzalez, 1988). The student questionnaire contained 15 sections, of which four were designed to measure students' ATM. Each section on ATM included three items that measured how important, difficult, and enjoyable students felt about each of the mathematical areas mentioned above. Students responded to each attitudinal item on a 5-point Likert scale. Attitudinal measures on algebra, geometry, and trigonometry were used, whereas those on arithmetic were not used in this study. Because the target population was high school seniors, arithmetic was considered less relevant.

Therefore, three sets of data (algebra, geometry, and trigonometry) were used in this study. Father's education level, mother's education level, student's sex, and student's ATM and AIM measures were the variables. Parent education levels, student's sex, and measures of student's AIM were identical across the data sets. Attitudinal measures, however, were different from data set to data set. Two statistical procedures were used in this study. First, the data set on algebra was used to derive the final model. The model development was both theory driven and data driven. The other two data sets on geometry and trigonometry were used to test the model derived from a literature review of previous research and data exploration on the algebra data set. If the model could satisfactorily fit the geometry and trigonometry data, confidence in the validity of the model would be greater. If there were serious problems on the model-data-fit for the two data sets, separate models would be developed.

Model Specification

The reciprocal model of the relationship between ATM and AIM is described in Figure 2. The model structure contained three blocks, beginning with father's education (FAED), mother's education (MOED), and student's sex (SEX), followed by student's ATM measures of algebra, geometry, or trigonometry indicated as important (IM), difficult (DI), and enjoyable (EN), and ending with student's AIM. Single arrows represent direct causal effects; the arrows point from the cause to the effect. Double arrows represent general correlations between two variables, assuming no causal implications.

Figure 2. The Structural Model of the Reciprocal Relationship Between ATM and AIM



In the first block, FAED, MOED, and SEX were allowed to correlate with one another (Ethington & Wolfle, 1984; Hayduk, 1987) (see left portion of Figure 2). FAED and MOED indicate not only parent education, but also parent ability and attitude (Scarr & Weinberg, 1978), and family background variables are influential in the learning of mathematics (e.g., Meece et al., 1982; Tsai & Walberg, 1983). The expectation was that students of parents with higher levels of education would achieve higher AIM and show more positive ATM (Ethington & Wolfle, 1984). FAED and MOED, therefore, were specified to affect both IM and AIM. On the other hand, there are no evident reasons to expect FAED and MOED to affect DI and EN. The reason is that, although some parents with higher levels of education can better help their children in the study of mathematics, the feeling of DI and EN in mathematics is largely personal and more directly affected by individual characteristics of students. This is one of the reasons why student's SEX was specified to affect, among other things, DI and EN. The relationship of SEX to other factors in the model was consistent with previous studies and reviews on sex differences in ATM and AIM (e.g., Aiken, 1970a, 1976; Ethington & Wolfle, 1984, 1986; Hyde, Fennema, & Lamon, 1990; Sherman, 1980).

The second block in the center of the model included attitudinal measures of IM, DI, and EN. IM was considered an awareness or a recognition; DI and EN were regarded as students' real feelings that could either encourage or frustrate them in their learning of mathematics. Based on these considerations, IM was treated as independent of DI and EN. Moreover, one may reasonably assume that DI directly influences EN, whereas EN has little direct impact on DI. The residuals of IM, DI, and EN were allowed to covary, however, assuming that these factors could share some common errors in their measurement.

The attitudinal block, in conjunction with the third block of the model (see right portion of Figure 2), specified the reciprocal relationships between ATM and AIM. This specification reflects the longstanding assumption in the research literature on the relationship between ATM and AIM (See Neale, 1969).

Most of the causal paths in the model were derived from literature review and theoretical assumptions except for the path from AIM to IM. There are no evident reasons to suggest that AIM has direct impact on IM. Once again, IM was considered an awareness or a recognition, independent of AIM. Thus, no specification was made for this path in the original model, which was added as a result of data exploration. The addition of this path to the original model substantially improved the model-data-fit. Nevertheless, this data driven decision in the model specification was tested for validation on the other two data sets and demonstrated to be feasible.

Statistical Analysis

Structure equation modeling has been recommended for testing theories and models in behavioral sciences (e.g., Anderson & Gerbing, 1988; Bentler, 1980; Hayduk, 1987; Keeves, 1986) for this manner of modeling controls for joint and reciprocal effects; it also takes measurement errors into account. Furthermore, structure equation modeling maintains the advantages of ordinary least squares regression that is useful to probe causal hypotheses and rule out rival causes (Mosteller & Tukey, 1977).

There were seven variables in the model. FAED, MOED, and SEX were exogenous variables that affected other variables, but received no effect of any kind. IM, DI, EN, and AIM were endogenous variables that might affect one another and might be affected by exogenous variables. A single indicator was used for each variable in the model. I used Linear Structural Relations (LISREL 7) (Jöreskog & Sörbom, 1988) to obtain the model parameter estimates and to test the goodness of the model-data-fit. Significance of the model parameters were tested through t- values. A t- value is a ratio of a maximum likelihood estimate to its standard error and therefore equivalent to a z- test. Standardized coefficients were tested for significance at the .05 level.

Along with Bollen (1989), I used multiple methods, including χ^2, goodness-of-fit index (GFI), adjusted goodness-of-fit index (AGFI), and root mean square residual (RMSR) to test the model-data-fit. The structural models estimated by maximum likelihood were fitted to a data set and compared for their model-data-fit. Alternative structural models included (a) the partial null model with no relations specified among variables (but with the measurement model specified); (b) the original model; (c) the revised model in which the path from AIM to IM was added to the original model (Figure 2). The partial null model was the base-line model against which other models were assessed. The original model was established on the basis of theoretical review of literature discussed earlier. The revised model added one more path to the original model according to the model-data-fit feedback from the LISREL program.

There are three loops in the model in Figure 2. Loop enhancement was, therefore, taken into account, and calculated as $1/(1 - L)$, where L is the coefficient product of those paths that form a causal loop (Hayduk, 1987). The modified direct or indirect effect was the product of the normal effect and the loop enhancement $1/(1 - L)$.

Some researchers tend to delete the nonsignificant paths from their hypothesized structural model. This deletion has been criticized because it compromises the model χ^2 test (See Hayduk, 1987). Nonsignificant standardized coefficients were maintained in the model in this study. A standardized coefficient indicates the change in a variable resulting from a unit change in one of its causal variables. The change is expressed in the form of standardized score that can be transformed easily into the original scale in which the variable is measured as long as its mean and standard deviation are available. In this manner, standardized coefficients can also be examined from the perspective of practical meaningfulness. I made no attempt in this study to perform significance-prompted model revision.

Results

Table 1 lists the three correlation matrices for the algebra, geometry, and trigonometry data sets. LISREL analyses in this study were based upon their corresponding variance–covariance matrices.

Model Testing and Revision

As mentioned earlier, the algebra data set was first used to derive and revise the model on the ATM–AIM relationship. A series of nested structural models were tested and compared to determine the best-fit model. Note that there was a sharp decrease in chi-square from the null model to the original model, suggesting that the specified paths were effective in explaining the variance–covariance matrix.

The original model did not include the path from AIM to IM, and its initial results of model-data-fit were not immediately acceptable. The modification index provided by the LISREL program indicated that the model-data-fit could be improved significantly if the path from AIM to IM was added to the original model. The revised model was reassessed through LISREL, and it fitted the data well. This procedure was then imposed on the geometry and trigonometry data sets. Table 2 summaries the model-data-fit statistics on those structural models for all three data sets.

Cross-validation of the revised model demonstrated relatively good results. Four different methods were used to assess the model-data-fit for the three data sets. Chi-squares were not statistically significant at the .05 level. Both GFIs and AGFIs were extremely close to the value of 1. RMSRs were reasonably trivial. All these indicated a relatively good model-data-fit for each data set. This consistency suggests that high school seniors had a similar structure on the causal ATM–AIM relationship across the three mathematical areas.

Model Interpretation

Standardized coefficients of parameters in the revised model for the algebra data set are presented in Table 3. The direct effect of DI on EN was statistically significant, indicating that DI was the most significant contributor to EN. The effects of IM on AIM, EN on AIM, and AIM on IM were also noteworthy. Although not statistically significant, they were substantially large from the practical point of view. IM and EN, therefore, were potential candidates that

Table 1.—Correlation Matrices of Three Data Sets ($N = 1{,}044$)

	1	2	3	4	5	6	7
Algebra							
1. Important	1.000						
2. Difficult	0.348	1.000					
3. Enjoyable	0.494	0.636	1.000				
4. AIM	0.005	0.125	0.076	1.000			
5. Father's education	0.001	–0.014	–0.025	0.119	1.000		
6. Mother's education	–0.057	0.027	–0.020	0.100	0.509	1.000	
7. Sex	–0.011	–0.027	–0.018	–0.202	–0.007	–0.037	1.000
Mean	4.24	3.45	3.78	18.71	3.02	2.81	1.62
SD	1.01	1.24	1.06	8.13	1.68	1.61	0.54
Geometry							
1. Important	1.000						
2. Difficult	0.356	1.000					
3. Enjoyable	0.522	0.680	1.000				
4. AIM	0.005	0.026	0.018	1.000			
5. Father's education	–0.021	0.002	–0.050	0.119	1.000		
6. Mother's education	–0.071	0.015	–0.060	0.100	0.509	1.000	
7. Sex	–0.071	–0.051	–0.073	–0.202	–0.007	–0.037	1.000
Mean	4.17	3.49	3.67	18.71	3.02	2.81	1.62
SD	0.93	1.17	1.17	8.13	1.68	1.61	0.54
Trigonometry							
1. Important	1.000						
2. Difficult	0.375	1.000					
3. Enjoyable	0.510	0.633	1.000				
4. AIM	–0.035	0.059	0.007	1.000			
5. Father's education	0.003	0.019	–0.031	0.119	1.000		
6. Mother's education	–0.023	–0.015	–0.071	0.100	0.509	1.000	
7. Sex	–0.012	–0.050	–0.053	–0.202	–0.007	–0.037	1.000
Mean	4.11	3.19	3.50	18.71	3.02	2.81	1.62
SD	0.99	1.27	1.24	8.13	1.68	1.61	0.54

Note. AIM = achievement in mathematics.

Table 2.—Model-Data-Fit Statistics of the Nested Models for Three Data Sets ($N = 1{,}044$)

Model	χ^2	*df*	*p*	GFI	AGFI	RMSR
Algebra						
Null	940.66	18	.000	0.804	0.452	0.569
Original	33.88	4	.000	0.991	0.989	0.519
Revised	2.53	3	.471	0.999	0.999	0.013
Geometry						
Null	863.64	18	.000	0.815	0.481	0.680
Original	46.20	4	.000	0.988	0.986	0.480
Revised	0.58	3	.902	1.000	1.000	0.007
Trigonometry						
Null	948.17	18	.000	0.804	0.452	0.593
Original	31.19	4	.000	0.992	0.990	0.466
Revised	4.98	3	.173	0.999	0.998	0.032

Note. Significance level was .05. GFI = goodness of fit index; AGFI = adjusted goodness of fit index; RMSR = root mean square residual.

Table 3.—Standardized Maximum Likelihood Estimates of Parameters of Revised Model for the Algebra Data Set ($N = 1{,}044$)

Effect variable	Cause variable Father's education	 Mother's education	 Sex	 Important	 Difficult	 Enjoyable	 AIM
Important	–0.162	–0.316	0.144	—	—	—	2.476
Difficult	—	—	0.001	—	—	—	0.093
Enjoyable	—	—	–0.001	—	0.640*	—	–0.119
AIM	0.292	–0.104	–0.078	–2.929	0.012	1.548	—

Note: AIM = achievement in mathematics.
$^*p < .05$.

might well be important contributors to AIM. The effect of AIM on IM might be useful in educational practice as well.

In the revised model (Figure 2), there are three loops between ATM and AIM. The results of loop enhancement are summarized in Table 4. The IM–AIM loop enhancement was .121. The modified direct effect of IM on AIM was –.355, indicating that the loop actually weakened the negative effect of IM on AIM. On the other hand, the modified direct effect of AIM on IM was .300. The loop, therefore, also weakened the positive effect of AIM on IM. In sum, the IM–AIM loop substantially constrained both the effect of IM on AIM and that of AIM on IM. The same analysis was applied to the EN–AIM loop that had a weakening effect with a loop enhancement of .845. The modified direct effect of EN on AIM was 1.308, and that of AIM on EN was –.101. The weakening effect of the EN–AIM loop (15%) was, therefore, not as extreme as that of the IM–AIM loop (88%).

In contrast to the IM–AIM and EN–AIM loops, the DI–AIM loop enhancement was 1.001, indicating a trivial strengthening effect. The modified direct effect of DI on AIM and that of AIM on DI remained almost the same as their normal effects. Note that DI also affected AIM indirectly via EN. This indirect effect of DI on AIM, influenced by the EN–AIM loop, was .541. The total effect of DI on AIM was .553, which might indicate that DI was a practically important contributor to AIM.

Table 5 summarizes the standardized coefficients of parameters in the revised model for the geometry data set. Standardized coefficients indicated that IM was the most significant contributor to AIM, followed by EN and FAED in that order; DI was the most meaningful contributor to EN; MOED was the most important contributor to IM. DI did not possess any significant contributors. The effects of both SEX and AIM on DI were neither statistically significant nor practically informative.

The effects of the IM–AIM and EN–AIM loop enhancement in the geometry data set were the same as those in the algebra data set (Table 4). However, the DI–AIM loop enhancement was .986. In comparison with the DI–AIM loop in the algebra data set, this loop had a slight weakening effect. The modified direct effect of DI on AIM was .115 (normal direct effect was .117), and that of AIM on DI was –.114 (normal direct effect was –.116). Therefore, with such a trivial loop modification, the interpretation was similar to that in the algebra data set.

Table 4.—Loop Enhancement in Revised Model for Three Data Sets

Loop	Data set Algebra	 Geometry	 Trigonometry
Important—AIM	0.121	0.194	0.310
Difficult—AIM	1.001	0.987	1.004
Enjoyable—AIM	0.845	0.841	0.748

Note: AIM = achievement in mathematics.

As shown in Table 6, for the trigonometry data set, IM was the most significant contributor to AIM, followed by EN and FAED in that order. DI was the most meaningful contributor to EN. No variable was found to have a detectable impact on DI. These were the same as found in the geometry data set. In addition, AIM was the most significant contributor to EN. AIM and MOED in that order were the most important contributors to IM.

The effects of loop enhancement in the trigonometry data set were much the same as those in the algebra and geometry data sets (Table 4). This consistency in the effects of loop enhancement indicates that the reciprocal relationship between ATM and AIM was similar in the three mathematical areas among high school seniors.

The model also had some informative bearings on sex differences in mathematics education. As observed in Tables 3, 5, and 6, SEX had no significant effect on any endogenous variables. This indicates that both boys and girls had similar feelings in regard to how important, difficult, and enjoyable mathematics was; sex differences were trivial in AIM as well.

Table 5.—Standardized Maximum Likelihood Estimates of Parameters of Revised Model for the Geometry Data Set ($N = 1{,}044$)

Effect variable	Father's education	Mother's education	Sex	Important	Difficult	Enjoyable	AIM
	Cause variable						
Important	–0.153	–0.242*	0.077	—	—	—	1.770
Difficult	—	—	–0.032	—	—	—	–0.116
Enjoyable	—	—	–0.022	—	0.690*	—	–0.141
AIM	0.141*	–0.030	–0.085	–2.347*	0.117	1.344*	—

Note: AIM = achievement in mathematics.
*$p < .05$.

Table 6.—Standardized Maximum Likelihood Estimates of Parameters of Revised Model for the Trigonometry Data Set ($N = 1{,}044$)

Effect variable	Father's education	Mother's education	Sex	Important	Difficult	Enjoyable	AIM
	Cause variable						
Important	–0.066	–0.215*	0.064	—	—	—	1.252*
Difficult	—	—	–0.030	—	—	—	–0.152
Enjoyable	—	—	–0.028	—	0.703*	—	–0.256*
AIM	0.170*	0.029	–0.044	–1.781*	–0.026	1.315*	—

Note: AIM = achievement in mathematics.
*$p < .05$.

Discussion

In the first block of the model, FAED and MOED were specified to affect both IM and AIM. The effect of FAED on AIM was significant, and so was the effect of MOED on IM. Fathers tend to have more influences on their children's achievement, whereas mothers tend to have more influence on their children's attitude. These findings from the Dominican data are in line with those from the American data (Ethington & Wolfle, 1984), suggesting that developing countries may share some commonalities with developed countries in regard to the relationship of parent education to mathematics attitude and achievement.

Furthermore, FAED had positive effects on their children's AIM: the higher the father's education level, the higher his child's mathematics achievement. This is another similarity between the Dominican data and the American data (Ethington & Wolfle, 1984). On the other hand, MOED had negative effects on their children's IM: the higher the mother's education level, the lower her child's perception of mathematics as important in one's life. One explanation is that mothers with a lower education level are more aware, perhaps from their educational and occupational frustrations, of the importance of mathematics and more frequently convey this recognition to their children. The American data, however, demonstrated positive effects of mother's education on mathematics attitude (Ethington & Wolfle). Therefore, the relationship between parental education and mathematics attitude may be more culturally diverse than the relationship between parent education and mathematics achievement.

Sex differences in the revised model were trivial for both ATM and AIM measures. SEX, as an exogenous variable, was specified to affect all four endogenous variables of IM, DI, EN, and AIM. There were no detectable sex differences in the model, however; sex differences were minor both statistically and substantially. All current meta-analyses of American studies demonstrated that sex differences are not pronounced in both mathematics attitude and mathematics achievement (Friedman, 1989, 1994; Frost, Hyde, & Fennema, 1994; Hyde et al., 1990). Thereafter, developing countries may also share some commonalities with developed countries with respect to sex differences in mathematics.

The second and third blocks in the revised model in which the reciprocal relationship between attitudinal measures and AIM was described are the central focus in this study. In the ATM block, the effect of DI on EN was significant across the three data sets. This finding provides evidence to the general intuition or assumption long existing among mathematics educators that if students feel that learning mathematics is difficult, they can hardly enjoy it. Many students are then at risk of dropping out of mathematics courses. To prevent this from happening, instructional measures ought to ensure that mathematics is presented in an interesting and attractive way. Meanwhile, the data supported the specification that EN had no direct effect on DI, indicating that the feeling of enjoyment does not necessarily ease the feeling of difficulty when learning mathematics.

IM was specified as independent of other attitudinal measures in the revised model, and this specification was not disproved by the data. It seems reasonable to assume that IM is a kind of awareness or recognition, an attitudinal element that may encourage students to put more effort into learning mathematics, but rarely affect other related attitudinal elements such as the feelings of difficulty and enjoyment. The implication is that successful efforts in bringing students to a better awareness of the importance of mathematics may not automatically improve other attitudinal aspects. On the other hand, either frustration or enjoyment in the learning of mathematics is unlikely to change the recognition of mathematics as an important discipline. Therefore, as Aiken (1970b) reported, one should be careful when using generic measures of attitude toward mathematics because even negative attitudes may contain some positive elements, such as the recognition of the importance of mathematics.

One of the most revealing findings in this study is the reciprocal relationship between each attitudinal measure and AIM. Without the IM–AIM loop, AIM positively affected IM. A higher level of AIM caused a more positive recognition of mathematics as important. This normal effect was substantially weakened by the IM–AIM loop, however. Without the IM–AIM loop, IM negatively affected AIM. Students with a better recognition of mathematics as important were actually those with a lower level of AIM. This negative effect of IM on AIM was also substantially weakened by the IM–AIM loop. The weakening effect of the IM–AIM loop ranged from 69% to 88% across the three data sets and, therefore, was highly effective. Thereafter, the reciprocal relationship between IM and AIM substantively restricted their causal functions. This finding suggests that the unidirectional relationship between IM and AIM substantially overestimates the effects of IM on AIM and AIM on IM. For example, in Tables 3, 5, and 6, IM appears to have the strongest effect (though negative) on AIM. With the loop enhancement being considered, however, IM had much weaker effects than EN across the three data sets.

Without the EN–AIM loop, EN affected AIM substantially, indicating that students who experienced more enjoyment learning mathematics achieved higher scores in that subject. The effect of AIM on EN was practically trivial, however, suggesting that mathematics achievement did not substantially affect enjoyment in learning mathematics. The loop or interaction between EN and AIM demonstrated a small weakening effect, suggesting that the unidirectional relationship between EN and AIM slightly overestimates the effects of EN on AIM and AIM on EN. These findings remind mathematics educators that students with high mathematics achievement do not automatically enjoy learning the subject. Therefore, it is inappropriate to assume that high achievers in mathematics have few attitudinal problems. On the other hand, instructional measures that help students enjoy learning mathematics can make a difference in mathematics achievement.

DI seems to function largely through EN to affect AIM, but has no directly meaningful effect on AIM. Therefore, it is the feeling of enjoyment, not the feeling of difficulty, that directly affects mathematics achievement. However, the feeling of difficulty seems to be the single important force in shaping the feeling of enjoyment. These imply that making difficult mathematical content easy to learn is barely enough to improve mathematics achievement. It is more important to ensure that difficult mathematical content is presented in an interesting, attractive, and enjoyable way.

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